Optimizing Portfolio Allocation, Project Financing Deals and the Cost of Capital for the Entrepreneur

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Abstract

This paper studies the situation of an entrepreneur who considers financing her own investment project characterized by its non-tradability on the market, low divisibility and high risk. We rely on the investor's mean-variance framework and provide a theoretical model to identify the entrepreneur's optimal portfolio allocation and her cost of capital (hurdle rate). We find that the entrepreneur's optimal portfolio is related to the optimal portfolio of an unconstrained investor with a similar risk aversion. We develop the entrepreneur's optimal investment curve in the mean-variance framework and show that it is related to the investor's Capital Market Line. Our model provides a cost of capital for the entrepreneur that is positively influenced by her risk aversion, the project variance and the project size. In a second stage, we develop our model and include a bargaining game between the entrepreneur and the bank in order to add the determination of the optimal interest rate in the optimization problem to account for borrowing constraints. We find that the increase of the borrowing rate has only a small positive impact on the entrepreneur's cost of capital. We perform a numerical analysis on a realistic entrepreneurial situation and obtain realistic estimates of the entrepreneur's cost of capital. We also find that the entrepreneur is able to decrease her cost of capital when combining her project with other assets.

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1. Introduction

This research paper aims to model the entrepreneur's decision problem and identify her optimal portfolio choice when she considers investing in her own investment project. We provide a theoretical framework in order to identify the entrepreneur's optimal portfolio allocation, optimal project financing terms, and cost of capital (project's hurdle rate or required rate of return), given the entrepreneur's specific constraints described hereunder. The portfolio allocation choice and the project financing optimization schemes are combined so that the entrepreneur adopts a strategy that minimizes her cost of capital.

Our starting point is a realistic entrepreneurial problem. When confronted with the decision to finance her own investment project, the entrepreneur is in a different situation from the one of a rational risk-averse investor willing to allocate her wealth to a portfolio of assets traded on the financial market. The entrepreneur's project has to be differentiated from a traded stock. There exist three main differences which are not mutually exclusive.

First, the entrepreneur is able to invest in a project which, in the absence of the entrepreneur, is not accessible to other investors. Indeed, the entrepreneur might own particular information (such as IP, skills, qualified labor) about an investment project and be able to activate non-tradable assets that would be idle otherwise. Information is privately owned and is not shared with other economic agents. She is thus able to identify an investment project that these agents are not able to exploit. There is information asymmetry and the hypothesis of homogenous information is rejected.

Second, the project requires a large amount of investment outlay (by the entrepreneur) and cannot be divided into a large amount of small investments by many different investors (the hypothesis of perfect markets with atomistic investments does not hold). The low divisibility of the project is due to the entrepreneur's willingness to hold a significant share of control in her project, but also – and perhaps primarily – to informational asymmetry: the public does not have access to enough information to be confident in the project and, if the entrepreneur wants to access business angels, venture capitalists or other private equity investors, she needs to credibly signal that she is confident in her project's success. Hence, the entrepreneur needs to invest a considerable proportion of her wealth in her project and the project cannot be considered as part of the market portfolio of tradable assets. Even though the entrepreneur has access to traded assets to complete her portfolio allocation, this constrained concentration of wealth in the project considerably reduces her ability to optimally diversify her investment portfolio and she will most often need to borrow money. In order to reduce concentration risk and the borrowing needs, but also in order to share risks and add economic value to the project, the entrepreneur has the opportunity to resort to private equity modes of financing, more precisely to contract with a venture capitalist (not included in the current version of the paper) in order to share the investment but also the project's control and profits. It can be optimal since the entrepreneur is able to reduce risk and better diversify her portfolio.

Third, the entrepreneur's project is riskier than a traded stock. It has a higher volatility which increases the risk of the entrepreneur's overall portfolio allocation. It also has a higher failure probability which impacts the project's expected return and creates more uncertainty for the lenders (higher credit risk). Moreover, the entrepreneur's project is less liquid than a traded stock (higher liquidity risk). Therefore, the entrepreneur is subject to borrowing constraints which should also be related to her concentration risk. She is constrained on the amount that she can borrow as well as on the term (longer term due to lower liquidity) and on the cost of the loan (credit and term spread). For all these reasons, she cannot borrow infinite amounts at the risk-free rate, but she has to search for lenders and obtain a loan whose amount and rate depend on project, entrepreneur and lender characteristics.

As a consequence of these imperfections, the models frequently used in finance to identify the optimal portfolio allocation or the expected/required return of portfolios or assets such as the Efficient Frontier in Markowitz's Modern Portfolio Theory (1952), the Capital Market Line (CML) and the Security Market Line (SML) in the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965 & Mossin, 1966) are not adapted "as is" to the entrepreneur. The determination of the project's cost of capital (hurdle rate) cannot be performed by applying the traditional tools underlying the equilibrium risk-return relationship.

The entrepreneur's decision problem includes four investment and financing decisions which our model aims to optimize:

1) Determine the optimal portfolio allocation program (including some diversification in other assets) including the project or not;
2) Identify and negotiate the optimal financing deal terms given her bargaining power;
3) Identify the project's cost of capital. Optimizing the portfolio allocation and the financing terms should give entrepreneurs the opportunity to minimize their project's cost of capital and, therefore, increase their project's attractiveness and the proportion of projects worth financing;
4) Decide whether to invest in the project or not based on the project's cost of capital (hurdle rate or required rate of return).

In this research paper, we implement the entrepreneur's optimal portfolio choice through expected utility maximization with the inclusion of the aforementioned market imperfections, considering that all other standard assumptions of Modern Portfolio Theory are satisfied. Moreover, in a second stage, we include a bargaining game (maximization of a bargaining game function) between the entrepreneur and the financiers in order to determine the optimal project financing terms. We only consider lenders as available external financiers. As an output, we aim to identify the required rate of return of the investment project, i.e. the cost of capital or hurdle rate, as a function of the model determinants: the financial market conditions (expected return and risk of traded assets), the entrepreneur's risk aversion and bargaining power vis-à-vis the financier, and the project characteristics (expected return, risk and size).

We aim to determine the optimal portfolio allocation for the entrepreneur given her constraint to invest a certain amount in her project, the resulting optimal borrowed (or deposited) amount and (in the second model specification) the optimal loan (or deposit) rate maximizing the entrepreneur's utility function or (in the second model specification) an overall expected utility surplus function (bargaining game function) including the expected utility surplus of the entrepreneur and the expected utility surplus of the lender. Then, we aim to obtain the entrepreneur's cost of capital or the minimum return she should require to invest in her project (combined with other assets) if she wants to get at least as much utility as without investing in her project (investing in a portfolio of traded assets only). We also aim to construct the entrepreneur's constrained optimal investment curve as a function of the project expected return, risk and other model determinants, and to compare it to the portfolio choice of an unconstrained investor (on the Capital Market Line) who does not access the entrepreneurial project but has the same risk aversion level.

In our framework, the entrepreneur has to finance the entire investment outlay needed to start or develop her project. She can obtain some funding from the bank but she cannot share the investment with any other investor. The entrepreneur can combine her investment in the project with some investment in tradable assets in order to optimize her portfolio. Initially, the entrepreneur has a maximum bargaining power vis-à-vis the bank. This situation corresponds to the situation where there would be no bargaining game with the bank and the entrepreneur could borrow at a given rate (for example, the risk-free rate). Next, in the second model specification, in order to account for financing constraints to reflect the difficulty for entrepreneurs to obtain bank loans to finance their projects, we include the bargaining game in the model. We can compare the results obtained in the two different situations.

These theoretical developments enable us to perform a numerical analysis based on realistic data estimated from market data or measured in other studies, in order to obtain some approximation of the entrepreneur's optimal portfolio allocation, optimal loan rate and cost of capital in a realistic situation. We compare the entrepreneur's cost of capital when she combines her project with other assets and when she does not. We also measure the impacts of the borrowing constraints.

Consequently, an important objective of this research paper is to help entrepreneurs making right investment and financing decisions. Another objective is to develop an improved measure of a project's cost of capital for entrepreneurs. This will improve their selection of projects and could reduce failure rate.

In the second section, we review the existing literature on the entrepreneur's cost of capital. Third, we present our setup. Fourth, we develop our theoretical framework with risk-free borrowing and lending rate. Five, we include the bargaining game between the entrepreneur and the bank in our model. Sixth, we perform a numerical analysis. Finally, we conclude.

2. Literature review

Portfolio choice theory finds its roots in Markowitz's Modern Portfolio Theory (1952) whose main contribution is the Efficient Frontier. It has led to the development of the Capital Asset Pricing Model (CAPM) (Sharpe, 1964; Lintner, 1965 & Mossin, 1966) whose main results are the Capital Market Line (CML) and the Security Market Line (SML). Later, it has been shown that results similar to those of the CAPM can be obtained by considering that investors maximize a quadratic (or a mean-variance) utility function when making portfolio choices (Sharpe, 2007).

We can support our assumption that entrepreneurs need to invest a large proportion of their wealth in their project by the results of two other studies. Moskowitz and Vissing-Jorgensen (2002) report that American households investing in private equity have on average around 45% of their total wealth invested in that asset class. Moreover, Bitler, Moskowitz and Vissing-Jorgensen (2005) report that most (more than 70% of) American entrepreneur hold 100% of their company's shares and that, in most (more than 80% of) American private companies, a majority of shares is held by only one household.

Our paper contributes to a relatively scarce existing theoretical literature on the entrepreneur's portfolio choice and cost of capital. Several determinants of the entrepreneur's optimal portfolio and cost of capital have already been studied in the financial literature. Risk aversion has been shown to have an influence on the entrepreneur's cost of capital by Garvey's theoretical model (2001). He introduces the entrepreneur's risk aversion and adapts the mean-variance portfolio optimization framework to the entrepreneur's particular situation in order to obtain her optimal allocation and, in turn, her cost of capital. However, Garvey's framework does not account for the possibility for the entrepreneur to optimize her overall portfolio allocation with an allocation in risky tradable assets different from the traditional market portfolio, and to contract with a private equity sponsor. Another methodology for measuring the entrepreneur's cost of capital has been proposed by Meulbroeck (2001) and Kerins, Keiholm Smith & Smith (2004). They adopt the opportunity cost of capital technique to measure the entrepreneur's required rate of return. This consists in measuring the return of an investment in the market portfolio leveraged to the same risk as the venture (or as a portfolio of the venture and the market portfolio) given by the risk-return relation of the CML. Their model shows that the cost of capital increases with the entrepreneur's wealth commitment in the project and with the project's volatility. However, this model does not account for the risk aversion of the entrepreneur.

These approaches do not solve a long-lasting puzzle regarding the cost of capital of the entrepreneur. According to Kerins et al. (2004), the measured costs of capital are much too large compared to the returns yielded by venture capital projects, so that entrepreneurs would never have the incentive to start new ventures. They conclude that non-pecuniary benefits seem to play a role making up for foregone returns. Our paper develops a theoretical framework that provides a simple and intuitive approach to solve this puzzle.

In empirical studies, Mueller and Spitz-Oener (2006) show a positive relation between the owner/manager's ownership share and the company's profits, and Mueller (2010) shows a positive relation between the owner/manager's share of personal wealth invested in the company and the company's performance (return to equity). These authors explain those relations by an incentive effect for the entrepreneur and a higher required rate of return.

Finally, in order to include the entrepreneur's financing choices, François and Hübner (2011) develop a theoretical framework to study the contracting choices between an entrepreneur and venture capitalists featuring the entrepreneur's risk aversion, and project expected return, risk and size as main determinants. They identify optimal contract terms. However, their framework ignores the entrepreneur's portfolio allocation choice.

3. Setup

The first specification of the model consists in the maximization of the entrepreneur's utility function. The entrepreneur can borrow and lend at the risk-free rate and the influence of financiers is ignored, as well as the influence of the entrepreneurial project characteristics on the cost of borrowing.

We aim to find the optimal portfolio allocation for the entrepreneur given her specific constraint of having to finance her project with a significant proportion of her wealth. Practically, we aim to find the best allocation for a portfolio composed of the entrepreneur's project (constrained minimum weight), tradable risky assets and the risk-free asset. To identify the best allocation for the entrepreneur, we maximize a quadratic (mean-variance) utility function. The variables included in the model are therefore the entrepreneur's risk aversion, the project size and the project and other assets' expected return and risk.

As an outcome of this analysis, the best combination of the project and tradable risky assets is not (necessarily) the tangency point of the "traditional" CML (Sharpe Ratio maximizing tangency line) on the new unconstrained efficient frontier (including the project). This is due to the entrepreneur's investment constraint. Her optimal portfolio is therefore not (necessarily) that combination, which would be leveraged to respect the investment constraint, and is thus not (necessarily) situated on that "traditional" CML. The cost of capital is presumably lower than the one obtained if selecting that combination and leveraging it on the "traditional CML" to respect the project weight constraint.

We derive new entrepreneur-adapted efficient frontier and CML respecting the entrepreneur's constraint. The entrepreneur-adapted CML is called the entrepreneur's optimal investment curve (all combinations of the project, marketable risky assets and the risk-free asset maximizing the return for a given risk and respecting the constraint). It is not a straight line but a concave curve (in the mean/standard deviation and the mean/variance frameworks). In the absence of a corner solution, the tangency point between this curve and one of the convex utility curves provides the optimal allocation for the entrepreneur.

Once her optimal portfolio is known, the entrepreneur's cost of capital is obtained by identifying the project return that equalizes the utility of this optimal entrepreneurial allocation (including the entrepreneurial project) and the utility of the optimal allocation for a perfectly diversified investor (including no entrepreneurial project). Hence, we fully derive the entrepreneur's cost of capital as a function of her risk aversion, the project size and the project's and other assets' expected return and risk.

In a second stage, we include a bank in the model. The interest rate is not exogenously fixed but it is determined through a bargaining game between the entrepreneur and the bank. Therefore, the bank is able to obtain part of the project's return and to adjust the interest rate according to the project's risk-return profile and other model determinants.

This second specification of the model consists in the optimization of the bargaining game between the entrepreneur and the bank. The bargaining game function is maximized in order to find the optimal allocation for the entrepreneur and the optimal interest rate in function of the entrepreneur's bargaining power vis-à-vis the bank and the model determinants already used in the first specification of the model. From the entrepreneur's optimal total portfolio expected return, optimal total portfolio variance and optimal utility obtained with the optimal allocation and the optimal interest rate, the entrepreneur's cost of capital and the entrepreneur's optimal investment curve are obtained in the same way as in the first specification of the model.

4. Model with risk-free borrowing and lending

We represent the situation of a risk-averse entrepreneur with a risk aversion of γ who considers investing in her own entrepreneurial project of expected return r_π and variance σ_π^2. This project requires an investment outlay of K in proportion of the entrepreneur's wealth (normalized to 1). Due to favorable information asymmetry, the entrepreneur does not share the investment with other investors. Here, she also does not have the possibility to get the support and contract with a venture capitalist. The entrepreneur can combine her investment in her project with some investments in risky tradable assets in order to optimize her portfolio and maximize her utility. She can also invest or borrow at the risk-free rate. These risky tradable assets have expected returns represented by the vector R and a variance-covariance matrix $\sum$. r_π^e and R^e represent the entrepreneurial project excess expected return and the vector of risky tradable assets excess expected returns respectively. Covariances between the project and the tradable risky assets are represented by the vector Ω.

Since the weight of the project in the entrepreneur's optimal portfolio allocation is a constraint (w_π = K) and the allocation in the risk-free asset will result from the constraint of the sum of portfolio weights equal 1, we aim to find the optimal weights for the x risky marketable assets represented by the x*1 vector w_P*, maximizing the entrepreneur's utility. Once these optimal weights w_P* have been obtained, we have the entire composition of the entrepreneur's optimal portfolio and we can find the optimal portfolio return R_p^* and risk $\sigma^2{}_p^*$, the optimal utility U_p^* and the cost of capital for the entrepreneur.

4.1. The entrepreneur's utility function

The objective of the entrepreneur is to maximize her expected utility under her venture project weight constraint. Her utility can be characterized by a quadratic (mean-variance) expected utility function:

$$U_P = R_P - 0{,}5\,\gamma\,\sigma_P^2$$

Her portfolio return and variance, and therefore her utility, can be obtained with the following formulas:

$$R_P = r_f + K\,r_\pi^e + w_P\,R^e$$

$$\sigma_P^2 = K^2\,\sigma_\pi^2 + w_P \sum w_P + 2\,K\,w_P\,\Omega$$

$$U_p = r_f + K\,r_\pi^e + w_P\,R^e - 0{,}5\,\gamma\,[K^2\,\sigma_\pi^2 + w_P \sum w_P + 2\,K\,w_P\,\Omega]$$

4.2. The entrepreneur's optimal portfolio

Therefore the entrepreneur wants to find the best allocation in risky marketable assets (w_P*) maximizing her utility. The optimal weights are obtained hereunder:

$$w_P{}^* = \frac{1}{\Upsilon} \Sigma^{-1}\,[R^e - \Upsilon K\,\Omega] = \frac{1}{\Upsilon} \Sigma^{-1}\,R^e - K \,\Sigma^{-1}\,\Omega$$

$$w_\pi^* = K \qquad\qquad w_{rf}^* = 1 - K - 1^T\,w_P{}^*$$

The entrepreneur's optimal allocation in risky marketable assets is related to the optimal allocation of an unconstrained perfectly diversified investor of similar risk aversion. Their optimal allocations increasingly differ as the project size rises. The optimal weights in the risky marketable assets for the entrepreneur are equal to those of the unconstrained investor minus an amount proportional to the project size and the covariance between the marketable assets and the project. They are thus over- or underweighted proportionally to the project size, depending on their negative or positive covariance with the project.

Entrepreneurs with a lower risk aversion will invest a greater part of their wealth in risky marketable assets and will therefore take more risk than more risk-averse entrepreneurs. From this optimal allocation, we can obtain the return and the variance of the optimal portfolio as well as its utility:

$$R_P^{*ent} = r_f + \frac{1}{\gamma} R^e \Sigma^{-1} R^e + K r_\pi^e - K R^e\,\Sigma^{-1}\Omega = R_P^{*inv} + K(r_\pi^e - R^e \Sigma^{-1}\Omega)$$

$$\sigma_P^{2*ent} = \frac{1}{\gamma^2} R^e \Sigma^{-1} R^e + K^2(\sigma_\pi^2 - \Omega\,\Sigma^{-1}\Omega) = \sigma_P^{2*inv} + K^2(\sigma_\pi^2 - \Omega\,\Sigma^{-1}\Omega)$$

Logically, the return and variance of the entrepreneur's optimal portfolio are also related to the optimal portfolio return and variance of an unconstrained perfectly diversified investor of similar risk aversion. The optimal return for the entrepreneur is equal to the optimal return of the unconstrained investor plus an amount positively related to the size and the return of the project and negatively related to the covariances of the project with the other assets. The optimal variance for the entrepreneur is equal to the optimal variance of the unconstrained investor plus an amount positively related to the square of the project size and to the project variance and negatively related to the covariances of the project with the other assets.

$$U_P^{*ent} = U_P^{*inv} + K\left(r_\pi^e - R^e\,\Sigma^{-1}\Omega\right) - 0{,}5\gamma K^2\left(\sigma_\pi^2 - \Omega\ \Sigma^{-1}\Omega\right)$$

Consequently, the entrepreneur's optimal utility is also related to the optimal utility of an unconstrained perfectly diversified investor of similar risk aversion. We have to compare those two optimal utilities in order to find whether it is optimal for the entrepreneur to invest in her project or whether she should better give up her project on financial return/risk criteria. Moreover, in a return/risk framework, the set of portfolios maximizing return for each level of risk for the entrepreneur is also linked to the set of the unconstrained investor (Capital Market Line of slope equal to the Sharpe ratio of the market portfolio). Those two points are developed in the next two sections.

4.3. <u>The entrepreneur's cost of capital</u>

The unconstrained investor only considers investing in marketable assets. Consequently, the utility of this investor can be represented as hereunder:

$$U_P = R_P - 0{,}5\,\gamma\,\sigma_P^2 = r_f + w_P\,R^e - 0{,}5\,\gamma\,w_P\,\Sigma\,w_P$$

The investor allocates his portfolio w_P with the aim of maximizing his utility. We can thus obtain the investor's optimal allocation w_P* and his optimal utility:

$$w_P{}^* = \frac{1}{\gamma}\,\Sigma^{-1}\,R^e \qquad\qquad w_{rf}^* = 1 - 1^T\,w_P{}^*$$

$$U_P^* = r_f + \frac{1}{2\gamma}\,R^e\ \Sigma^{-1}R^e = r_f + \frac{1}{2\gamma}\,SR_m^2$$

We can find the entrepreneur's cost of capital which is the minimum return that gives her a higher utility from investing in the optimal portfolio including her project than in the optimal portfolio of an unconstrained investor (not including the project):

$$k_\pi^{ent} = r_f + R^e\,\Sigma^{-1}\Omega + 0{,}5\,\gamma\,K\,(\sigma_\pi^2 - \Omega\,\Sigma^{-1}\Omega)$$

The entrepreneur's cost of capital is positively related to her risk aversion, the project risk (variance), the project size and negatively related to the covariances of the project with the other assets.

Comparison with the case without portfolio optimization

The entrepreneur's utility without other assets is obviously positively related to her project return and negatively related to her project variance. The entrepreneur has no allocation decision to take. She invests the needed amount K in her project and invests or borrows the rest of her wealth at the risk-free rate.

$$U = r_f + K\, r_\pi^e - 0{,}5\, \gamma\, K^2\, \sigma_\pi^2$$

Investing in marketable assets gives the opportunity to the entrepreneur to reduce her cost of capital. If the entrepreneur only considers investing in her project and in the risk-free asset, the entrepreneur's cost of capital is positively related to the project risk and to the Sharpe ratio of the market portfolio. However, the impact of the entrepreneur's risk aversion and the project size are uncertain.

$$k_\pi = \mathrm{r}_f + \frac{1}{2\,\gamma\,\mathrm{K}} R^e\, \Sigma^{-1} R^e + 0{,}5\,\gamma\,\mathrm{K}\,\sigma_\pi^2 = \mathrm{r}_f + \frac{SRm^2}{2\,\gamma\,\mathrm{K}} + 0{,}5\,\gamma\,\mathrm{K}\,\sigma_\pi^2$$

4.4. The entrepreneur's optimal investment curve (expected return-risk framework)

4.4.1. Indifference curves/lines

In the mean-variance framework, the entrepreneur's preferences are characterized by utility-indifference lines (see Figure 2 in appendix). The entrepreneur's utility-indifference lines are upward-sloping with a positive slope of ½ γ:

$$U_P = R_P - \frac{1}{2}\gamma\sigma 2_P <=> R_P = U_P + \frac{1}{2}\gamma\sigma 2_P \Rightarrow Sl = \frac{1}{2}\gamma$$

In the mean-standard deviation framework, the entrepreneur's preferences are characterized by utility-indifference curves (see Figure 4 in appendix). The entrepreneur's utility-indifference curves are upward-sloping with a positive tangency slope of $\gamma\ \sigma_P$:

$$Sl = \frac{R_P}{\sigma_P} = \gamma\ \sigma_P$$

4.4.2. The entrepreneur's optimal investment curve

We have seen that there is a relation between the optimal allocation in marketable assets of the entrepreneur and that of an unconstrained perfectly diversified investor with a similar risk aversion. Consequently, there is also a relation between the return, the variance and the utility of their optimal portfolios. Furthermore, there is also a relation between the set of possible optimal portfolios for the entrepreneur and the one for the perfectly diversified investor

(Capital Market Curve) in the mean-variance framework. The Capital Market Curve of the investor is represented by the following well-known portfolio return-risk equation:

$$R_P = r_f + SR_m \sqrt{\sigma_P^2} = r_f + SR_m\, \sigma_P$$

The relation between the optimal return and the optimal variance for the investor can also be expressed as hereunder:

$$R_P = r_f + \gamma\, \sigma_P^2$$

By using the relations between the optimal portfolio return and risk of the entrepreneur and the investor, we can obtain an equivalent relation for the entrepreneur's best available return-risk portfolios (the entrepreneur's optimal investment curve):

$$R_P = r_f + K\left(r_\pi^e - R^e\, \Sigma^{-1}\Omega\right) + SR_m \sqrt{\sigma_P^{2*} - K^2(\sigma_\pi^2 - \Omega\, \Sigma^{-1}\Omega)}$$

As for the investor, the relation between the optimal return and the optimal variance for the entrepreneur can also be expressed as hereunder:

$$R_P = r_f + K\left(r_\pi^e - R^e\, \Sigma^{-1}\Omega\right) + \gamma\, [\sigma_P^{2*} - K^2(\sigma_\pi^2 - \Omega\, \Sigma^{-1}\Omega)]$$

We obtain the equivalent of the Capital Market Curve for the entrepreneur which includes her constrained investment in her project. A representation of the curve is given in Figure 1 in appendix. The curve is an upward-sloping concave curve containing all portfolios which maximize return for each level of available risk for the entrepreneur given her investment constraint. The entrepreneur's portfolio return is an increasing concave function of her portfolio variance or standard deviation. In the return-standard deviation framework, this is different from the linear relation between the portfolio return and the portfolio standard deviation in the case of the unconstrained investor, represented by the Capital Market Line with a slope equal to the Sharpe ratio of the market portfolio.

Given the convexity of the entrepreneur's utility curves, the entrepreneur's optimal portfolio is necessarily situated on the entrepreneur's optimal investment curve. As for the CLM of the perfectly diversified investor, the entrepreneur's curve is independent of her risk aversion. Therefore, all entrepreneurs with a project of similar characteristics have the same optimal investment curve and their risk aversion determines which portfolio they will choose among all portfolios situated on this curve.

The slope of the tangency to one point of the entrepreneur's optimal investment curve is given by the first derivative of the curve function:

$$Sl = \frac{R_P}{\sigma_P^2} = \frac{SR_m}{2\sqrt{\sigma_P^2 - K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\,\Omega]}}$$

Similarly, the slope of the tangency to one point of the Capital Market Curve is given by the first derivative of the curve function:

$$Sl = \frac{R_P}{\sigma_P^2} = \frac{SR_m}{2\sqrt{\sigma_P^2}}$$

The slope of the tangency to the entrepreneur's curve is the same as the slope of the tangency to the Capital Market Curve for a variance difference of $K^2\,[\sigma^2_\pi - \Omega\,\sum^{-1}\Omega]$. The concavity of the entrepreneur's optimal investment curve and the investor's curve are also related. They are identical with the same variance difference (variance of the entrepreneur's minimum-variance optimal portfolio):

$$\frac{\frac{SR_m}{2\sqrt{\sigma_P^2 - K^2(\sigma_\pi^2 - \Omega\sum^{-1}\Omega)}}}{\sigma_P^2} = -\frac{SR_m}{\sigma_P^2 - K^2(\sigma_\pi^2 - \Omega\sum^{-1}\Omega)} \text{ (entrepreneur)}$$

$$\frac{\frac{SR_m}{2\sqrt{\sigma_P^2}}}{\sigma_P^2} = -\frac{SR_m}{\sigma_P^2} \text{ (investor)}$$

For that variance difference, the entrepreneur's curve has the same slope and curvature as the perfectly diversified investor's Capital Market Curve. Therefore, the entrepreneur's curve is simply a translation of the Capital Market Curve. At any variance level available for the entrepreneur, for a same variance level, the two curves have a return difference of $K\,[r_\pi^e - R^e \sum^{-1}\Omega]$. At any return level available for the entrepreneur, for a same return level, the two curves have a variance difference of $K^2\,[\sigma^2_\pi - \Omega\,\sum^{-1}\Omega]$. As shown by the optimality formulas, there will always be the same return $K\,(r_\pi^e - R^e \sum^{-1}\Omega)$ and risk $K^2\,(\sigma_\pi^2 - \Omega\sum^{-1}\Omega)$ differences between the optimal portfolios of an entrepreneur and an unconstrained investor of similar risk aversion, whatever their risk aversion. For a same variance level and at any of them, the entrepreneur's curve has a higher slope and a higher concavity than the curve of an investor with a similar risk aversion. Therefore, the entrepreneur's curve intersects the investor's curve at a certain variance level and then the entrepreneur's return keeps on increasing at a higher speed.

The slope and the concavity of the curves are important to identify the optimal portfolios depending on the entrepreneur and the investor's risk aversion. The intersection point between the entrepreneur and the investor's curves does not correspond to the same entrepreneur and investor's risk aversions. Therefore, an entrepreneur and an investor of similar risk aversion never choose the same portfolio return and variance.

The entrepreneur's optimal investment curve will be raised (higher return for a given risk) when the project return and the market portfolio's Sharpe ratio are higher, and when the project variance is lower, since a higher return can be obtained at a lower risk. Therefore, the entrepreneur will have the possibility to raise her utility. The effect of the project size depends on the return and variance.

Since the entrepreneur and a perfectly diversified investor of similar risk aversion are characterized by the same utility-indifference lines, the entrepreneur will decide to invest in her project (combined with marketable assets) if the tangent utility line to the entrepreneur's curve is situated above the tangent utility line to the Capital Market Curve of the perfectly diversified investor.

When her risk aversion decreases below a certain level (all other things constant), the slope of her utility lines decrease and it becomes more interesting for the entrepreneur to invest in her project:

$$\sigma_P = \frac{K\ [(r_\pi^e - R^e \sum^{-1} \Omega)^2 + {SR_m}^2(\sigma_\pi^2 - \Omega \sum^{-1} \Omega)]}{2\ SR_m\ (r_\pi^e - R^e \sum^{-1} \Omega)}$$

The higher the risk and the size of her project, the less risk averse the entrepreneur has to be for investing in her project.

4.4.3. Minimum-variance portfolio

Due to her investment constraint, the entrepreneur cannot reduce the risk of her investment to 0, contrary to the unconstrained investor. She is forced to invest K in her project. The entrepreneur's curve does not start from the zero risk level. However, she can reduce her risk below $K^2\ \sigma^2_\pi$ by investing in other assets whose correlations with her project are negative. Consequently, the minimum-variance portfolio, which will be chosen by very risk-averse entrepreneurs, has a variance between 0 and $K^2\ \sigma^2_\pi$.

The min-variance portfolio can be obtained by running the entrepreneur's utility maximization with an extremely high level of risk aversion. Moreover, this result is also obtained by considering an infinite risk aversion in the previously defined formulas for the optimal portfolio return and variance for the entrepreneur. The variance of the min-variance portfolio also corresponds to the minimum variance for which the entrepreneur's curve function exists. The min-variance portfolio has the following return and risk levels:

$$R_{minP} = r_f + K[r_\pi^e - R^e \textstyle\sum^{-1}\Omega]$$

$$\sigma_{minP}^2 = K^2[\sigma_\pi^2 - \Omega \textstyle\sum^{-1} \Omega]$$

The lower the variance and the size of her project, the more the entrepreneur has the possibility to reduce the risk of her portfolio.

4.4.4. Tangency portfolio

The entrepreneur will select one of all portfolios situated on her optimal investment curve, depending on her risk aversion. The optimal portfolio for the entrepreneur is situated at the tangency point of one of the utility-indifference lines on the entrepreneur's curve (return/variance framework) (see Figure 2 in appendix) or of one of the utility-indifference curves on the entrepreneur's curve (return/standard deviation framework) (Figure 4), corresponding to the highest accessible utility level. Therefore, the optimal portfolio for the entrepreneur will correspond to the risk level for which the slope of the tangency to the entrepreneur's curve is equal to the slope of the utility lines (mean/variance framework) or to the slope of the tangency to the utility curves (mean/standard deviation framework). We obtain the same portfolio as the portfolio obtained by directly maximizing the entrepreneur's utility:

Slope of the tangency to the entrepreneur's curve $= \frac{R_P}{\sigma_P^2} = \frac{SR_m}{2\sqrt{\sigma_P^2 - K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]}}$

$<=> \frac{1}{2}\gamma = \frac{SR_m}{2\sqrt{\sigma_P^2 - K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]}} \Leftrightarrow \sigma_P^{2*} = \frac{1}{\gamma^2}\ SR_m^2 + K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]$

We can do the same in the return-standard deviation framework:

Slope of the tangency to the entrepreneur's curve $= \frac{R_P}{\sigma_P} = \frac{SR_m\ \sigma_P}{\sqrt{\sigma_P^2 - K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]}}$

$<=> \gamma\sigma_P^* = \frac{SR_m\ \sigma_P^*}{\sqrt{\sigma_P^2 - K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]}} \Leftrightarrow \sigma_P^{2*} = \frac{1}{\gamma^2}\ SR_m^2 + K^2[\sigma_\pi^2 - \Omega\ \Sigma^{-1}\ \Omega]$

We can follow the same process for the unconstrained investor's optimal portfolio on the Capital Market Curve. As expected, the portfolio variance equalizing the slope of the utility-indifference lines and the slope of the tangency to the Capital Market curve coincides with the variance of the investor's optimal portfolio:

$$R_P = r_f + SR_m\sqrt{\sigma_P^2}$$

$$\frac{1}{2}\gamma = \frac{SR_m}{2\sqrt{\sigma_P^2}} \Leftrightarrow \sigma_P^{2*} = \frac{1}{\gamma^2}\ SR_m^2$$

The slope of the tangency to the entrepreneur's curve decreases as the portfolio variance increases. Depending on her risk aversion, the entrepreneur chooses the portfolio maximizing her utility among the portfolios situated on the curve. Entrepreneurs with a high risk aversion (steep utility lines/curves) prefer a portfolio on the left (steeper) side of the curve characterized by low return and risk, whereas entrepreneurs with low risk aversion (lowly sloped utility lines/curves) prefer a portfolio situated on the right (less steep) side of the curve characterized by high return and risk.

4.4.5. Impact on the curves and the entrepreneur's decision of variations in the parameters

A modification in the project return only moves the entrepreneur's curve. An increase in the project return translates the entrepreneur's curve up. It increases the return of the minimum variance optimal portfolio. However, it does not modify its slope and curvature which remain the same for each variance level. It modifies only the return of the entrepreneur's optimal portfolio, not its variance. The entrepreneur's optimal portfolio is only moved up. The investor's curve is not affected by the project characteristics.

If the project return is insufficient, the entrepreneur's optimal portfolio corresponds to a lower utility level than the investor's optimal portfolio. The tangent utility line to the entrepreneur's curve is below the tangent utility line to the investor's curve. In order to obtain the entrepreneur's cost of capital, we aim to find the project return which raises the entrepreneur's curve in order to make the two curves tangent to the same utility line. This situation corresponds to equal optimal utility levels for the entrepreneur and the investor. If the project return is higher than the entrepreneur's cost of capital, the entrepreneur's optimal portfolio corresponds to a higher utility level than the investor's optimal portfolio. The tangent utility line to the entrepreneur's curve is above the tangent utility line to the investor's curve. The entrepreneur has the incentive to invest in her project together with her optimal allocation in other assets.

A modification in the project variance only moves the entrepreneur's curve, not the investor's curve. An increase in the project variance translates the entrepreneur's curve to the right. It does not modify the overall form of the curve but its slope and curvature for each variance level are increased. Therefore, it increases the variance of the entrepreneur's optimal portfolio, but it does not change its return. The entrepreneur's optimal portfolio is only moved to the right. This new optimal portfolio is situated on a utility line corresponding to a lower utility level. The higher the project variance, the higher the project return necessary to make the entrepreneur's optimal portfolio situated on the same utility line as the investor's optimal portfolio. Consequently, it confirms that the entrepreneur's optimal utility is negatively related to the project variance and that the entrepreneur's cost of capital is positively related to the project variance.

A modification in the project size only moves the entrepreneur's curve, not the investor's curve. An increase in the project size translates the entrepreneur's curve up and to the right. It does not modify the overall form of the curve but its slope and curvature for each variance level are increased. Therefore, it increases both the return and the variance of the entrepreneur's optimal portfolio. Whether the new optimal portfolio is situated on a utility line corresponding to a higher or a lower utility level depends on the return and the variance of the project. The sensitivity of the entrepreneur's optimal utility to the project size is given by:

$$\frac{U^{e*}}{K} = r_\pi^e - R^e \ \Sigma^{-1}\, \Omega - \gamma\, K(\sigma_\pi^2 - \Omega\, \Sigma^{-1}\, \Omega)$$

Consequently, the impact of a modification of the project size on the entrepreneur's cost of capital also depends on the other project characteristics. A decrease in the entrepreneur's utility leads to an increase in her cost of capital. However, it can happen that an increase in the entrepreneur's optimal utility still leads to an increase in her cost of capital. The sensitivity of the entrepreneur's cost of capital on the project size is given by the following expression which is (nearly) always positive:

$$\frac{k_\pi}{K} = 0.5\,\gamma\,(\sigma_\pi^2 - \Omega \textstyle\sum^{-1} \Omega)$$

Consequently, the entrepreneur's cost of capital is positively related to the project size.

5. Model including a bargaining game with the bank

In the current version of the second model specification, we do not let the entrepreneur choose the composition of the portfolio of tradable assets. She can compose her allocation only with the constrained quantity K in her project π and a quantity w_P in a portfolio of tradable assets whose composition is given, and borrow or deposit at the interest rate c determined with the bank in the bargaining game. As seen with our first model specification, the portfolio of tradable assets is not the investor's market portfolio since we can show that, due to her constraint of investing a constrained quantity in her project, the entrepreneur would not choose the market portfolio and her project as her optimal allocation. This model specification can be further developed in order to give the entrepreneur the possibility to optimize the composition of her allocation in tradable assets, but it does not modify the obtained relations between the model determinants and the optimized variables.

For situations involving additional parties (a bank in this version of the paper), the overall expected utility surplus from the deal (bargaining game function) has to be maximized. Consequently, the variables that we aim to optimize from the model are:

- The entrepreneur's allocation weight (w_P) in the portfolio of tradable assets;
- The loan (or deposit) amount (it is not really an additional variable since it results from the allocation weights);
- The loan (or deposit) rate (c).

As an output, the minimized project's cost of capital (minimum project return necessary for the entrepreneur to have as much utility when investing in the project in the optimal way given by the model as when investing in the best combination of the market portfolio and the risk-free asset) will also be obtained in function of the model determinants. It serves as the decision criterion for the entrepreneur to invest or not in the project by comparison with the project's expected return. The entrepreneur's optimal investment curve including her bargaining power will also be obtained. The variables are optimized in function of several determinants from the literature: the entrepreneur's risk aversion (γ), all contractors (entrepreneur, bank)' bargaining power (η, $1-\eta$), the project's size (K), expected return (r_π) and variance (σ_π^2), the bank's opportunity cost of debt (r^b) (minimum required loan rate when the entrepreneur borrows), the expected return (r_P) and variance (σ_P^2) of the portfolio of tradable assets and the covariance between the project and the portfolio of tradable assets ($Cov(r_\pi; r_P)$).

5.1. The bargaining game function

As in the first model specification, the entrepreneur's expected utility function is a quadratic (mean-variance) utility function:

$$U^{ent} = K\, r_{\pi} \;+ w_P\, r_P \;+ (1 - K - w_P)\, c$$

$$-0.5\, \gamma\, \left(K^2 \sigma_{\pi}^2 + w_P^2\, \sigma_P^2 + 2\, K\, w_P\, Cov(r_{\pi}\; ; r_P\;)\right)$$

In the bargaining game, the entrepreneur's utility is represented by her excess expected utility U^{ent}-U^i where U^i is the utility that the entrepreneur could obtain if she would not consider investing in her project and would behave as an investor by choosing the optimal portfolio of tradable assets only.

The bank's expected utility function (U^b) is a linear function of the loan (or deposit) amount and the loan (or deposit) interest rate (c). The (excess) expected utility of the bank corresponds to the bank's excess expected return since the bank has a perfectly diversified portfolio of loans which mitigates its risk. It is an excess utility function since the utility is positive only if the bank grants loans or pays deposits at better financing terms than the bank's opportunity cost of financing (r^b). The bank is able to lend at higher rates than the market and to pay deposits at lower rates thanks to its bargaining power. We represent the bank's (excess) expected utility function (U^b) by the following equation:

$$U^b \;= (K + w_P - 1)\, (c - r^b)$$

The bank's opportunity cost of debt (r^b) is the interest rate the bank can expect to obtain by financing other (equivalent) projects. It is the minimum acceptable rate to finance the project. The bank requires a minimum rate under which it will not accept lending money to the entrepreneur. In the current specification of the model, this rate has to be determined by the model user. This rate may depend on the characteristics of the project. In some cases, the entrepreneur can be willing to short sell the portfolio of tradable assets. This does not directly mean that she does not need to borrow money since the shortsale may not compensate for the large amount invested in the project. In cases where the entrepreneur has money available for a deposit at the bank, the interest rate is lower than the borrowing rate and is capped by the maximum acceptable deposit rate for the bank.

If the entrepreneur's bargaining power is equal to 1, the entrepreneur has a complete control on the determination of the optimal choices. She can borrow at the minimum rate required by the bank for accepting the loan and, therefore, the bank's excess expected utility is equal to 0. It corresponds to the case where the entrepreneur maximizes her utility function, as in the first specification of our model, except that the minimum rate required by the bank replaces the risk-free rate and is, in most cases, higher.

If the bank's bargaining power is equal to 1, the bank has a complete control on the determination of the optimal choices. She can charge a higher interest rate which is capped by the constraint that the entrepreneur has to obtain a utility similar to the utility she would obtain by investing in tradable assets only. Otherwise, she would not accept investing in her project.

For intermediate cases, the bargaining game function has to be maximized where η is the entrepreneur's bargaining power:

$$G = (U^{ent} - U^{i})^{\eta} \, (U^{b})^{1-\eta}$$

To represent total expected utility surplus (G) with two parties, François and Hübner (2011) use a similar bargaining game solution, which was proposed by Fan and Sundaresan (2000), including the expected utility surpluses of the entrepreneur and the venture capitalist and their bargaining power. We replace the investor by the bank. The proportion of the overall expected utility surplus obtained by each contractor depends on his bargaining power.

The entrepreneur's bargaining power can take any value between 0 and 1. If the entrepreneur's bargaining power is maximal (equal to 1), the bargaining game function corresponds to the entrepreneur's excess expected utility and the maximization problem is equivalent to the initial specification of our model without bargaining game with the bank, except that the interest rate is the bank's opportunity cost of debt which may not be equal to the risk-free interest rate.

5.2. The entrepreneur's optimal portfolio

The entrepreneur's optimal portfolio does not depend on her bargaining power. The optimal allocation is only impacted (decreased) if the bank's opportunity cost of debt is larger than the risk-free interest rate. The influence of the other parameters remains the same.

$$w^{*ent} = \frac{\frac{1}{\Upsilon}(r_P - r^b) - K\, Cov(r_\pi; r_P)}{\sigma_P^2}$$

The entrepreneur's optimal borrowing needs are increasingly larger when the optimal allocation in the portfolio of tradable assets increases.

$$K + w^{*ent} - 1 = K - 1 + \frac{\frac{1}{\Upsilon}(r_P - r^b) - K\, Cov(r_\pi; r_P)}{\sigma_P^2}$$

5.3. The optimal interest rate

The optimal interest rate is positively related to the bank's opportunity cost and the project's expected return. Moreover, it is negatively related to the entrepreneur's bargaining power, the project's variance and the entrepreneur's risk aversion.

Interest rate: $c = r^b + (1-\eta)\, \frac{K\, r_\pi + (1-K)\, r^b - 0.5\,\gamma\, K^2 \sigma_\pi^2 - U^i + \frac{0.5}{\gamma\, \sigma_P^2}(r_P - r^b - \gamma\, K\, Cov(r_\pi\,; r_P\,))^2}{K - 1 + \frac{1}{\gamma\, \sigma_P^2}\,(r_P - r^b - \gamma\, K\, Cov(r_\pi\,; r_P\,))}$

For a given bargaining power, the interest rate is increasingly higher as the expected return of the project increases and its variance decreases. This is normal since the bank is able to obtain a higher return if the project is better. However, we can expect that the bank adjusts the interest rate in function of the risk it takes by financing the project.

The optimal interest rate does not have the expected relations to all determinants included in the model. The result obtained for the optimal interest rate can be considered as surprising and unrealistic. Since the model is a bargaining game between the entrepreneur and the bank and these two players will share the overall excess expected utility, it will be optimal for them that the bank requires a higher interest rate if the project has a higher expected return and a lower variance. Consequently, the optimal interest rate is negatively related to the risk-return profile of the project. However, we may expect that the bank adjusts the interest rate to account for the risk-return profile of the project. In order to correct for this limitation of our model, the minimum interest rate required by the bank could depend on the project's risk-return profile. We can also believe that the entrepreneur has less bargaining power if her project has a less attractive risk-return profile. Therefore, the bargaining power determinant included in the model may be adjusted and the model may be further developed to account for the relation between the bargaining power and the project characteristics.

The bargaining power can also be determined endogenously as a function of the project's expected return and risk and the other model determinants, so that the interest rate is adjusted in function of the risk taken by the bank. An entrepreneur would have less bargaining power if her project has a less attractive risk-return performance and this reduction of the bargaining power increases the interest rate.

We can endogenously determine the entrepreneur's bargaining power as a function of the project's expected return and variance by using the optimized relation of the interest rate. We can expect that the entrepreneur has more bargaining power if her project has a higher expected return and a lower variance. By isolating the entrepreneur's bargaining power in the relation of the interest rate, we obtain such relations. We can use a fixed value of c in order to obtain an estimation of the bargaining power. This value can then be used in the model formulas. By fixing the value of the interest rate, we can express the bargaining power as a function of the project characteristics. With this adjusted value of the bargaining power, we can take into account the risk included in the project. The bank adjusts the interest rate according to the risk to which it is exposed from the risk-return profile of the project. The interest rate is adjusted upwards for riskier projects (due to the reduction of the bargaining power coming from the worse risk-return profile) and downwards for less risky projects (due to the increase of the entrepreneur's bargaining power).

5.4. <u>Optimal utilities</u>

The entrepreneur's optimal utility is positively related to her bargaining power, the project's expected return and the expected return of tradable assets. It is negatively related to the project's variance, the entrepreneur's risk aversion, the variance of tradable assets and covariance of the project with tradable assets.

$$U^{ent*} = \eta \left[K\, r_\pi \;\; + (1-K)\, r^b - 0.5\,\gamma\, K^2 \sigma_\pi^2 + \frac{0.5}{\gamma\, \sigma_P^2}\left(r_P - r^b - \gamma\, K\, Cov(r_\pi; r_P)\right)^2\right] + (1-\eta)\, U^i$$

The bank's optimal utility is negatively related to the entrepreneur' bargaining power, the project's variance, the entrepreneur's risk aversion, the variance of tradable assets and covariance of the project with tradable assets. Moreover, it is positively related to the project's expected return and the expected return of tradable assets.

$$\text{Bank}U^{b*} = (1-\eta)\, [K\, r_\pi \;\; + (1-K)\, r^b - 0.5\,\gamma\, K^2 \sigma_\pi^2 - U^i + \frac{0.5}{\gamma\, \sigma_P^2}\left(r_P - r^b - \gamma\, K\, Cov(r_\pi; r_P)\right)^2]$$

5.5. The entrepreneur’s cost of capital

The entrepreneur’s cost of capital does not depend on her bargaining power. It is only impacted (increased) by the higher bank’s opportunity cost of debt compared to the risk-free rate. The impact of the other parameters remains the same.

$$k_\pi \;= \left(1 - \frac{1}{K}\right) r^b + 0.5\,\gamma\, K\, \sigma_\pi^2 + \frac{U^i}{K} - \frac{0.5}{\gamma\, K\, \sigma_P^2}\,(r_P - r^b - \gamma\, K\, Cov(r_\pi; r_P))^2$$

5.6. The entrepreneur’s optimal investment curve (expected return-risk framework)

The optimal return of the entrepreneur depends on her bargaining power whereas her optimal variance does not depend on her bargaining power:

$$R^{ent*} = \eta \left[K\, r_\pi \;\; + (1-K)\, r^b - \frac{(r_P \;-\; r^b)\, K\, Cov(r_\pi; r_P)}{\sigma_P^2}\right] + \frac{0.5\,(1+\eta)\,(r_P - r^b)^2}{\gamma\, \sigma_P^2} + (1-\eta)\left[U^i + 0.5\,\gamma\, K^2\, (\sigma_\pi^2 - \frac{Cov(r_\pi; r_P)^2}{\sigma_P^2})\right]$$

$$\sigma^{2\;ent*} = \frac{1}{\gamma^2\, \sigma_P^2}(r_P - r^b)^2 + K^2\, (\sigma_\pi^2 - \frac{Cov(r_\pi; r_P)^2}{\sigma_P^2})$$

The entrepreneur’s optimal investment curve depends on her bargaining power. For a given level of total variance, the entrepreneur is able to obtain a higher total return if she has a higher bargaining power. With a maximal entrepreneur's bargaining power, a curve similar to the case without bargaining game is obtained.

Entrepreneur's investment curve: $R^{ent} = \eta \left[K\, r_\pi + (1-K)\, r^b - \frac{(r_P - r^b)\, K\, Cov(r_\pi; r_P)}{\sigma_P^2}\right] + \frac{0.5\,(1+\eta)\,(r_P - r^b)\,\sqrt{\sigma^{2\,ent} - K^2\,(\sigma_\pi^2 - \frac{Cov(r_\pi;r_P)^2}{\sigma_P^2})}}{\sqrt{\sigma_P^2}} + (1-\eta)\left[U^i + \frac{0.5\,K^2\left(\sigma_\pi^2 - \frac{Cov(r_\pi;r_P)^2}{\sigma_P^2}\right)(r_P - r^b)}{\sqrt{\sigma_P^2}\sqrt{\sigma^{2\,ent} - K^2\,(\sigma_\pi^2 - \frac{Cov(r_\pi;r_P)^2}{\sigma_P^2})}}\right]$

6. Numerical analysis

We select parameter values as much realistic as possible by using values similar to those estimated or used in the paper of François and Hübner (2011). For our base case, we choose an entrepreneur's risk aversion of 1, a project's size of 2 (200% of the entrepreneur's initial wealth), a project's annual expected return of 30%, a project's annual standard deviation of 35% and a beta of 1.25 between the project and the portfolio of tradable assets. The bargaining power is perfectly divided between the entrepreneur and the bank: a bargaining power of 0.5 for the entrepreneur and for the bank. The bank's opportunity cost of debt is 5% annually and the risk-free interest rate is 3% annually. We also choose an annual expected return of 12% and an annual standard deviation of 16% for the portfolio of tradable assets.

Base case parameters	
rf	0.03
γ	1
K	2
rπ	0.3
σπ	0.35
η	0.5
κb	0.05
rp	0.12
σp	0.16
Covπp	0.032
βπp	1.25
Corrπp	0.571
Ui	0.188

With our base case application, we obtain an optimal allocation for the entrepreneur of 0.23 in the portfolio of tradable assets. Therefore, the entrepreneur has to borrow 1.23 (123% of her initial wealth) from the bank. She borrows that amount at an interest rate of 9.76% annually. The entrepreneur has to require an annual return of 24.12%.

Base case results	
wp	0.234
c	0.098
kπ	0.241
Ue	0.247
Ue-Ui	0.059
Ub	0.059
K+wp-1	1.234
Ue+Ub	0.306
Ue-Ui+Ub	0.118

We compare the situations corresponding to the different specifications of our model. The results are available in appendix from Table 2 to Table 7. First, the entrepreneur has to enter a bargaining game with the bank (with the base case parameter values). Second, the entrepreneur has a bargaining power of 1 (maximum) in the bargaining game (the other parameters keep the base case values). Third, the entrepreneur can borrow or lend at the risk-free rate (the other parameters keep the base case values). For these three situations, we also compare the case where the entrepreneur diversifies (and optimizes her allocation) by investing in the portfolio of tradable assets with the case where she invests in her project only. We can see that the entrepreneur's cost of capital does not change a lot.

The entrepreneur's cost of capital does not depend on her bargaining power. This is, in the first place, due to the non-influence of the bargaining power on the entrepreneur's portfolio allocation decision. Whatever her bargaining power and the result of the bargaining game about the interest paid by the entrepreneur, the rate which impacts the optimal allocation in the portfolio of tradable assets is always the minimum interest rate required by the bank. This is also because, whatever the result of the bargaining game about the interest rate paid by the entrepreneur, the interest rate which impacts the entrepreneur's cost of capital is always the minimum interest rate required by the bank to finance. The spread over that rate determines the sharing of the overall excess utility between the two players, but it has no influence on their decisions to invest in (finance) the project or not: the entrepreneur requires the same expected return from her project for taking the decision to invest. In the situation of a relation between the entrepreneur and the bank for the determination of the optimal relations, having to borrow at a larger rate has no influence on the entrepreneur’s cost of capital and on her decision to invest or not in the project. The entrepreneur's cost of capital is influenced by the overall excess expected utility and not only by the entrepreneur's expected utility.

However, in the situation where the entrepreneur can borrow at a lower rate (for example, the risk-free rate) without having to enter a relation with a bank, she adapts her optimal allocation in the portfolio of tradable assets to this reduction of the borrowing rate and the interest rate which impacts the entrepreneur's cost of capital is this new rate. Therefore, in that situation, she is able to increase her expected utility and reduce her cost of capital. However, the difference is not huge. Consequently, having to borrow at a larger rate is not much detrimental for the entrepreneur. It only has a small influence on her cost of capital and, therefore, on her decision to invest or not in her project.

Moreover, the possibility to diversify her allocation by adding some investment in a portfolio of tradable assets also has only a small impact on her cost of capital. Diversification creates utility for the entrepreneur but only in a small extent and its influence on the entrepreneur's decision is weak.

Besides, the entrepreneur's bargaining power has a strong influence on the interest rate she has to pay and, therefore, on her utility, but, nevertheless, it has no influence on her cost of capital and on her decision to invest or not in her project.

7. Model including a bargaining game with the bank and with the venture capitalist

The third model specification involves a third player: the venture capitalist. Unfortunately, this third model specification is not described in detail yet in this (draft) version of the paper. We should be able to provide a more detailed description of this third model specification in a coming version of the paper.

This model specification involves three players: the entrepreneur (e), the venture capitalist (v) and the bank (b). The capital investment (K) needed to fund the entrepreneurial project has to be provided partly by the entrepreneur (S) and partly by the venture capitalist (K-S). The entrepreneur and the venture capitalist have to decide on the repartition of the investments. Besides, the entrepreneur can invest in risky assets traded on the market in order to diversify his portfolio allocation. The entrepreneur has to decide on the investment weights (w_P) in the risky assets traded on the market.

The entrepreneur can borrow some funds from the bank if his wealth (normalized to 1) is not sufficient to fund the optimal investments in the entrepreneurial project (S) and in the other risky assets (weights given by the vector w_P). Otherwise, the entrepreneur deposits the rest of his wealth at the bank. The bank and the entrepreneur have to decide on the interest rate (c). The interest rate (c) paid or received by the entrepreneur is determined in the bargaining game and depends on the players and project characteristics.

The shares of the project capital (and eventually the shares of the project return, profits, added value, capital value increase) obtained by the entrepreneur (d*S) and by the venture capitalist (K-d*S) are not necessarily equal to their investments (S and K-S) in the project. They can decide on a different repartition of the project capital shares compared to the investments. This is called dilution which is materialized by the dilution factor (d). The dilution factor (d) (via the transfer rate) is determined in the bargaining game and depends on the players and project characteristics. A dilution factor higher than 1 is advantageous for the entrepreneur whereas a dilution factor lower than 1 is beneficial for the venture capitalist.

In the model, dilution is not represented by the dilution factor (d) but by the transfer rate (τ). The dilution (corresponding to the transfer rate) can thereafter be obtained from the transfer rate. This transfer of money through the transfer rate is artificial as it does not directly happen in reality (it happens via the dilution but not directly through a loan). Thus, the transfer rate is the artificial interest rate at which the entrepreneur borrows from the venture capitalist (or lends to the venture capitalist) the same amount as the amount which the entrepreneur borrows from the bank (or deposit at the bank). The transfer rate is artificially added to the bank's interest rate but it concerns the venture capitalist and not the bank. The transfer rate can be positive or negative depending on whether the dilution is favorable or unfavorable to the entrepreneur and it also depends on the (borrowing or deposit/lending) situation. In a borrowing situation, a positive (negative) transfer rate corresponds to an unfavorable (a favorable) dilution for the entrepreneur $d<1$ ($d>1$) since the entrepreneur would borrow at a higher (lower) interest rate and the venture capitalist would receive the corresponding added part of the interest payments (support the bank's loss of interest payments). In a deposit/lending situation, a positive (negative) transfer rate corresponds to a favorable (an unfavorable) dilution for the entrepreneur $d>1$ ($d<1$) since the entrepreneur would deposit/lend at a higher (lower) interest rate and the venture capitalist would support the bank's loss due to the increase of interest payments (receive the corresponding difference in interest payments). The venture capitalist and the entrepreneur can decide on the dilution (or the transfer rate).

This model specification includes two additional parameters: the venture capitalist’s bargaining power (ε) and the venture capitalist’s cost of equity capital. The venture capitalist's cost of equity capital is the minimum acceptable return for funding similar entrepreneurial projects. In the end, the venture capitalist may accept a lower project return if the dilution is favorable since the additional return obtained from a favorable dilution may compensate for the lower project return. Hence, the project's cost of capital may be lower than the venture capitalist’s cost of equity capital.

The optimization problem involves the maximization of the bargaining game function (G). All three players obtain some benefits from funding the entrepreneurial project. If this is not the case, they do not accept funding the project.

$$G = \left(U^{ent} - U^{i}\right)^{\eta} \left(U^{b}\right)^{1-\eta-\varepsilon} \left(U^{v}\right)^{\varepsilon}$$

The project's cost of capital (or the entrepreneur’s cost of capital) is the project return required by the entrepreneur for accepting to fund his project. At the same time, this project's cost of capital as given by our model is also equivalent to the project return required by the venture capitalist and by the bank for accepting to fund the entrepreneurial project. The excess utilities obtained by the three players are a proportion (equivalent to their bargaining power) of the total excess utility. Therefore, the excess utilities of the three players are positive at the same time and negative at the same time (in the same situations). The entrepreneur's cost of capital has been reduced thanks to the combination of venture capital funding and bank financing.

8. Conclusion

We provide a framework to determine the entrepreneur's optimal portfolio and her cost of capital. In the first specification of our model, the entrepreneur is able to borrow or lend at the risk-free rate. We find that the entrepreneur's optimal portfolio is related to the optimal portfolio of an unconstrained investor with a similar risk aversion, and that the difference between these two optimal allocations is proportional to the project size. Consequently, we also find that the entrepreneur's optimal return, variance and utility are also related to those of an unconstrained investor with a similar risk aversion. We develop the entrepreneur's optimal investment curve in the mean-variance framework and relate it to the equivalent of the investor's Capital Market Line in the mean-variance framework. The entrepreneur selects her own optimal portfolio on her curve according to her risk aversion. We find that the entrepreneur's curve has exactly the same curvature and is simply the translation of the investor's curve. Our formula for the entrepreneur's cost of capital provides the expected relations with our different factors. The entrepreneur's risk aversion, the project variance and the project size have a positive impact on the entrepreneur's cost of capital.

The assumption of risk-free borrowing is not much realistic since entrepreneurial projects entail much risk for the financiers. Therefore, banks would not accept granting loans at the risk-free rate for such projects. We develop an advanced specification of our model in order to allow for higher borrowing interest rates. This specification proposes the optimization of a bargaining game between the entrepreneur and a bank. It optimizes not only the entrepreneur's portfolio allocation (and simultaneously the loan or deposit amount) but also the bank's interest rate in function of the bargaining power, by maximizing the bargaining game function including both players' excess expected utility functions and their bargaining power.

The entrepreneur's optimal allocation is not much impacted by the bargaining game compared to the initial situation without the bank. It does not depend on the bargaining power. It is slightly modified because the bank's opportunity cost of debt replaces the risk-free interest rate. The entrepreneur decreases her allocation in the portfolio of tradable assets due to this increase of the interest rate (borrowing cost).

The optimal interest rate is negatively related to the entrepreneur's bargaining power. It is equal to the bank's opportunity cost of debt if the entrepreneur has a maximal bargaining power and it is increasingly larger for lower values of the entrepreneur's bargaining power since the bank is able to obtain a higher return from the project when its bargaining power increases.

The advanced model also determines the entrepreneur's cost of capital: the return she should require for investing in her project. It shows that this cost of capital does not depend on the entrepreneur's bargaining power. The entrepreneur's cost of capital is not really impacted by the bargaining game compared to the initial situation without the bank. It is (slightly) increased because the bank's opportunity cost of debt replaces the risk-free interest rate.

With the advanced model, we can also obtain an advanced version of the entrepreneur's optimal investment curve which depends on the bargaining power. For a given level of total variance, the entrepreneur is able to obtain a higher total expected return if she has a higher bargaining power.

The entrepreneur's risk aversion and the project size in proportion of the entrepreneur's wealth are important to determine the entrepreneur's optimal allocation and cost of capital. The risk aversion, the project size and the project risk have a positive impact on the entrepreneur's cost of capital. Mueller (2010) found a positive relation between the share of entrepreneur's wealth invested in the project and the project return with a higher cost of capital or an incentive effect as possible explanations. Our model provides such relations. This is a different result from the required return given by the CAPM which does not depend on risk aversion and investment size.

Our study also highlights that the borrowing constraint to which entrepreneurs are subjected does not have a considerable impact on their optimal allocation and on the entrepreneur's cost of capital. The borrowing constraint increases the entrepreneur's cost of capital only in a small proportion, so that it does not have a strong influence on the attractivity of projects and on the entrepreneur's decision to invest or not.

At this stage, our framework does not include all possibilities for the entrepreneur to finance her project. The entrepreneur is not obliged to finance the totality of the project with her own wealth and bank loans. She has the possibility to contract with a venture capitalist in order to share the investment, obtain support but also share the profits and the decision-making of the project. Therefore, we aim to extend our framework by integrating this contracting relation between the entrepreneur and a venture capitalist. This will be added in an extension of our paper.

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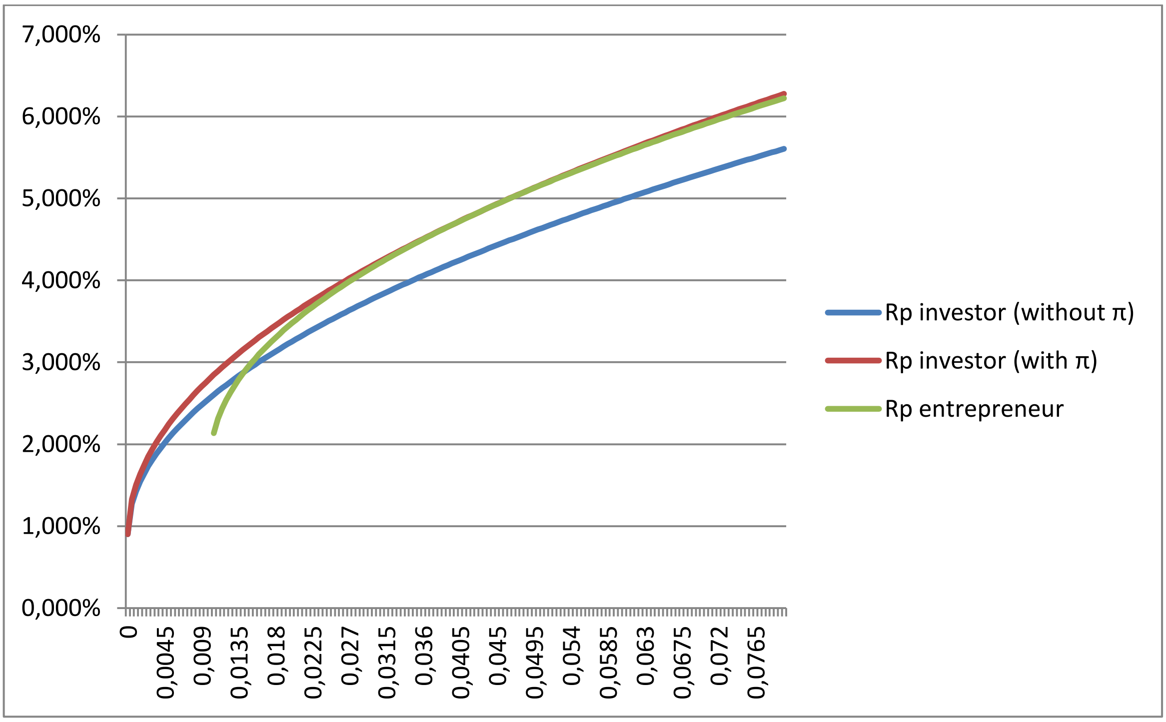
Appendix

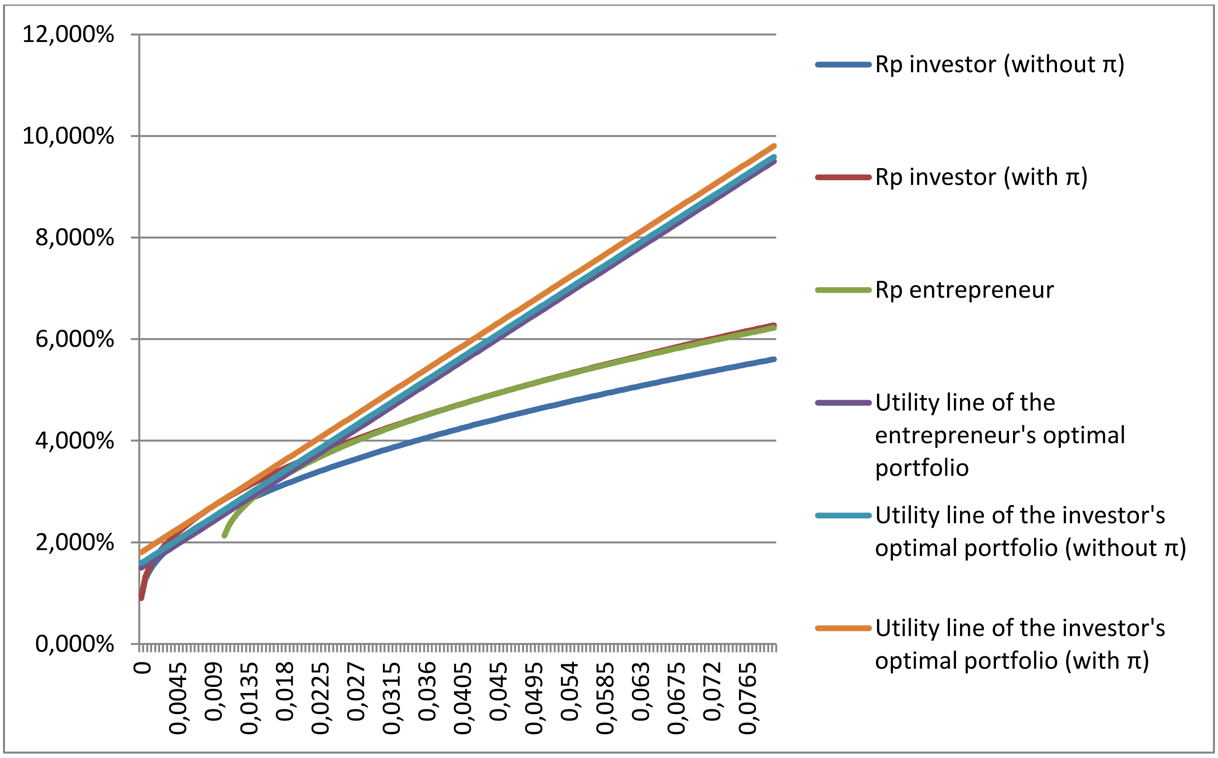
A) Figures

Figure 1: Optimal investment curves for the entrepreneur, the unconstrained investor without the project and the unconstrained investor with the project (mean/variance)



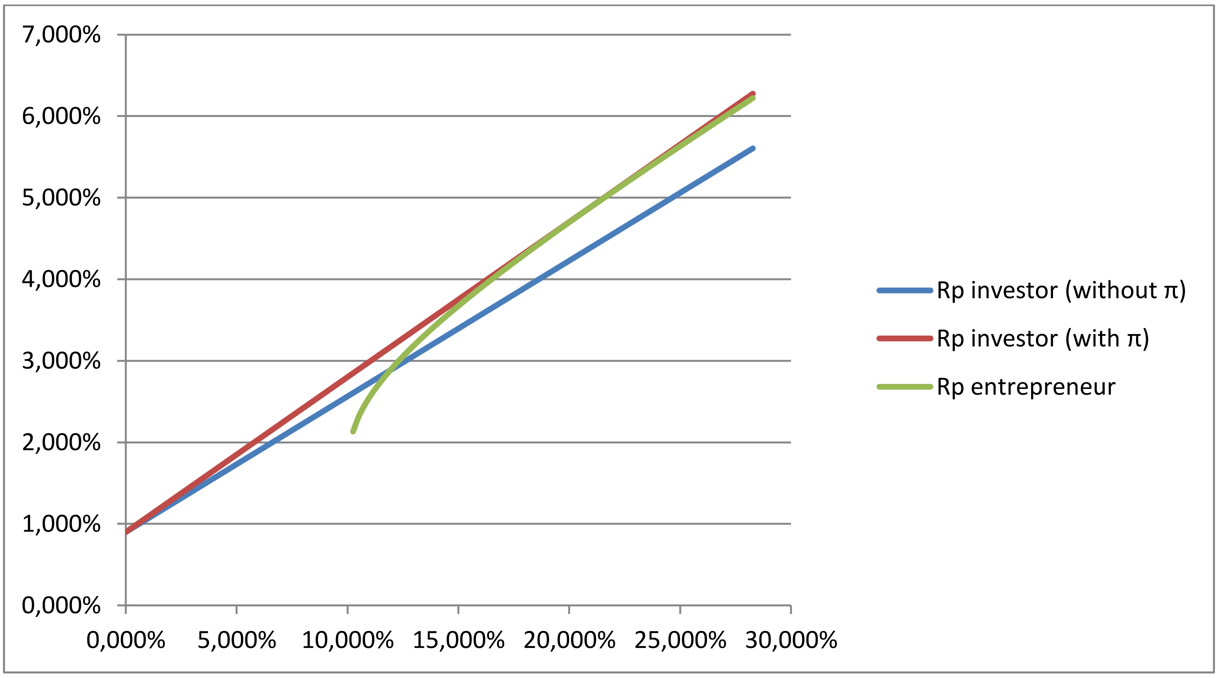
This figure represents the optimal investment curves for the entrepreneur, the unconstrained investor without access to the project and the unconstrained investor with access to the project in the expected return/variance framework, computed with the base case parameters and with the equilibrium expected returns and historical (co)variances of the marketable assets. For each optimal investment curve, the figure plots the (maximum available) expected portfolio return as a function of the portfolio variance.

Figure 2: Optimal investment curves with their tangent utility lines (mean/variance framework)



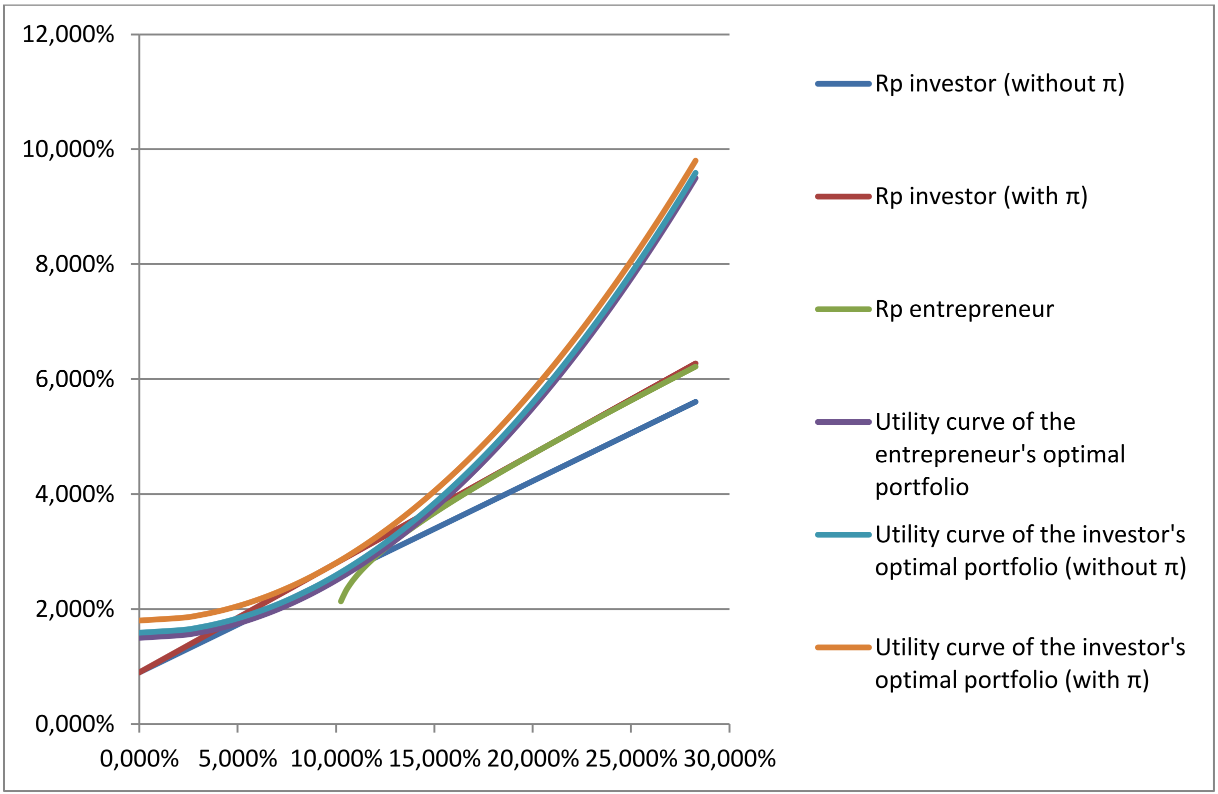
This figure represents the optimal investment curves for the entrepreneur, the unconstrained investor without access to the project and the unconstrained investor with access to the project in the expected return/variance framework, computed with the base case parameters and with the equilibrium expected returns and historical (co)variances of the marketable assets. Moreover, the figure also represents the tangent utility line (corresponding to the agent's highest available utility level) to each optimal investment curve, each tangency point giving the optimal portfolio of the corresponding agent.

Figure 3: Optimal investment curves/lines for the entrepreneur, the unconstrained investor without the project and the unconstrained investor with the project (mean/standard deviation framework)



This figure represents the optimal investment curves/lines for the entrepreneur, the unconstrained investor without access to the project and the unconstrained investor with access to the project in the expected return/standard deviation framework, computed with the base case parameters and with the equilibrium expected returns and historical (co)variances of the marketable assets. For each optimal investment curve/line, the figure plots the (maximum available) expected portfolio return as a function of the portfolio standard deviation.

Figure 4: Optimal investment curves/lines with their tangent utility curves (mean/standard deviation framework)



This figure represents the optimal investment curves/lines for the entrepreneur, the unconstrained investor without access to the project and the unconstrained investor with access to the project in the expected return/standard deviation framework, computed with the base case parameters and with the equilibrium expected returns and historical (co)variances of the marketable assets. Moreover, the figure also represents the utility curves corresponding to the highest available utility level of each agent. Each tangency point between the agent's optimal investment curve/line and his best available utility curve gives the optimal portfolio of the corresponding agent.

B) Tables

Table 1: Base case parameter values

Base case parameters	
rf	0.03
γ	1
K	2
rπ	0.3
σπ	0.35
η	0.5
κb	0.05
rp	0.12
σp	0.16
Covπp	0.032
βπp	1.25
Corrπp	0.571
Ui	0.188

Table 2: Optimal results for the base case (including the bargaining game with the bank)

Base case results	
wp	0.234
c	0.098
kπ	0.241
Ue	0.247
Ue-Ui	0.059
Ub	0.059
K+wp-1	1.234
Ue+Ub	0.306
Ue-Ui+Ub	0.118

Table 3: Optimal results for the base case without portfolio diversification (including the bargaining game with the bank)

No diversification	
wp	0
c	0.108
kπ	0.242
Ue	0.247
Ue-Ui	0.058
Ub	0.058
K+wp-1	1
Ue+Ub	0.305
Ue-Ui+Ub	0.117

Table 4: Optimal results for the base case if the entrepreneur has a maximal bargaining power (including the bargaining game with the bank)

η=1	
wp	0.234
c	0.05
kπ	0.241
Ue	0.306
Ue-Ui	0.118
Ub	0
K+wp-1	1.234
Ue+Ub	0.306
Ue-Ui+Ub	0.118

Table 5: Optimal results for the base case without portfolio diversification if the entrepreneur has a maximal bargaining power (including the bargaining game with the bank)

η=1 & no diversification	
wp	0
c	0.05
kπ	0.242
Ue	0.305
Ue-Ui	0.117
Ub	0
K+wp-1	1
Ue+Ub	0.305
Ue-Ui+Ub	0.117

Table 6: Optimal results for the base case with borrowing and lending at the risk-free rate

No bank => borrowing at rf	
wp	1.016
c	0.03
kπ	0.225
Ue	0.338
Ue-Ui	0.15
Ub	/
K+wp-1	2.016

Table 7: Optimal results for the base case with borrowing and lending at the risk-free rate without portfolio diversification

No diversification & borrowing at rf	
wp	0
c	0.03
kπ	0.232
Ue	0.325
Ue-Ui	0.137
Ub	/
K+wp-1	1